5. Factorization of Algebraic Expressions

Exercise 5.1

1. Question

Factorize:

$$x^3+x-3x^2-3$$

Answer

Given,

$$x^3 + x - 3x^2 - 3$$

$$= x(x^2 + 1) - 3(x^2 + 1)$$

$$=(x-3)(x^2+1)$$

2. Question

Factorize:

$$a(a+b)3-3a^{2}b(a+b)$$

Answer

Given,

$$a(a+b)^3 - 3a^2b(a+b)$$

$$= a(a+b)\{(a+b)^2 - 3ab\}$$
use the identity: $(a+b)^2 = a^2 + b^2 + 2ab$

$$= a(a + b)(a^2 + b^2 + 2ab - 3ab)$$

$$= a(a+b)(a^2+b^2-ab)$$

3. Question

Factorize:

$$x(x^3-y^3)+3xy-(x-y)$$

Answer

Given,

$$x(x^3-y^3)+3xy-(x-y)$$

As
$$(x^3 - y^3) = (x - y)(x^2 + xy + y^2)$$

$$x(x^3-y^3)+3xy-(x-y) = x[(x-y)(x^2+xy+y^2)] + 3xy-(x-y)$$

Take x(x-y) common to get,

$$x(x-y)[(x^2 + xy + y^2) + 3y]$$

4. Question

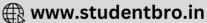
Factorize:

$$a^2x^2+(ax^2+1)x+a$$

Answer

$$a^2x^2 + (ax^2 + 1)x + a = a^2x^2 + ax^3 + x + a$$





$$= ax^{2}(a+x) + 1(x+a)$$

$$=(ax^2+1)(x+a)$$

Factorize:

$$x^2+y-xy-x$$

Answer

Given,

$$x^2 + y - xy - x = x^2 - x + y - xy$$

$$= x(x-1) + y(1-x)$$

$$= x(x-1) - y(x-1)$$

$$= (x-y)(x-1)$$

6. Question

Factorize:

$$x^3-2x^2y+3xy^2-6y^3$$

Answer

Given,

$$x^3 - 2x^2y + 3xy^2 - 6y^3$$

$$= x^{2}(x-2y) + 3y^{2}(x-2y)$$

$$=(x-2y)(x^2+3y^2)$$

7. Question

Factorize:

Answer

Given,

$$6ab-b^2+12ac-2bc = b(6a-b)+2c(6a-b)$$

$$= (b+2c)(6a-b)$$

8. Question

Factorize:

$$\left(x^2 + \frac{1}{x^2}\right) - 4\left(x + \frac{1}{x}\right) + 6$$

Answer

$$\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+6$$

$$= \left(x^2 + \frac{1}{x^2} + 2\right) - 4\left(x + \frac{1}{x}\right) + 4$$

$$= \left(x + \frac{1}{x}\right)^2 - 4\left(x + \frac{1}{x}\right) + 4$$





$$= \left\{ \left(x + \frac{1}{x} \right) - 2 \right\}^2$$
 [BY applying (a² - 2ab + b²) = (a-b)²]

Factorize:

$$x(x-2)(x-4)+4x-8$$

Answer

Given,

$$X(x-2)(x-4)+4x-8 = x(x-2)(x-4)+4(x-2)$$

$$= (x-2) (x(x-4)+4)$$

$$= (x-2) (x^2 - 4x + 4)$$

$$= (x-2) (x-2)^2$$

$$= (x-2)^3$$

10. Question

Factorize:

$$(x+2)(x^2+25)-10x^2-20x$$

Answer

Given,

$$(x+2)(x^2+25) - 10x^2 - 20x = (x+2)(x^2+25)-10x(x+2)$$

$$= (x+2) (x^2+25-10x)$$

$$= (x+2) (x-5)^2$$

11. Question

Factorize:

$$2a^2+2\sqrt{6}ab+3b^2$$

Answer

Given,

$$2a^2\sqrt{6ab+3b^2} = (\sqrt{2}a)^2 + 2(\sqrt{3} \times \sqrt{2})ab + (\sqrt{3}b)^2$$

$$= (\sqrt{2a} + \sqrt{3b})^2$$

12. Question

Factorize:

$$(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$$

Answer

$$(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$$

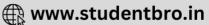
$$= (a-(b-c))^2 + (a+(b-c))^2 + 2(a-(b-c)) (a+(b-c))$$

[Applying identity:
$$x^2 + y^2 + 2xy = (x + y)^2$$
, where $x = a-(b-c)$, $y = a+(b-c)$]

$$=[(a-(b-c)) + (a+(b-c))]^2=(2a)^2=4a^2$$







Factorize:

$$a^{2}+b^{2}+2(ab+bc+ca)$$

Answer

Given,

$$a^2+b^2+2(ab+bc+ca)$$

$$= a^2+b^2+2ab+2bc+2ca$$

$$= (a+b)^2 + 2c(a+b)$$

$$= (a+b)(a+b+2c)$$

14. Question

Factorize:

$$4(x-y)^2-12(x-y)(x+y)+9(x+y)^2$$

Answer

Given.

$$4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2 = 4(x^2-2xy+y^2) - 12(x^2-y^2) + 9(x^2+y^2+2xy)$$

$$= 4x^2-8xy+4y^2-12x^2+12y^2+9x^2+9y+18xy$$

$$= x^2 + 25y^2 + 10xy$$

$$= (x)^2 + (5y)^2 + 2 \times x \times 5y$$

$$= (x+5y)^2$$

15. Question

Factorize:

$$a^{2}-b^{2}+2ab-c^{2}$$

Answer

Given,

$$a^2-b^2+2ab-c^2 = a^2-(b^2-2bc+c^2)$$

$$= a - (b-c)^2$$

$$= (a + (b-c)) (a-(b-c))$$

$$= (a+b-c) (a-b+c)$$

16. Question

Factorize:

$$a^2+2ab+b^2-c^2$$

Answer

Given,

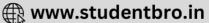
$$a^2+2ab+b^2-c^2 = (a+b)^2 - c^2$$

$$= (a+b-c)(a+b+c)$$

17. Question







Factorize:

$$a^2+4b^2-4ab-4c^2$$

Answer

Given,

$$a^2 + 4b^2 - 4ab - 4c^2 = (a)^2 + (2b)^2 - 2 \times a \times 2b - 4c^2$$

$$= (a-2b)^2 - (2c)^2$$

$$= (a-2b-2c)(a-2b+2c)$$

18. Question

Factorize:

$$xy^9-yx^9$$

Answer

Given,

$$xy^9 - yx^9 = xy (y^8 - x^8)$$

$$= xy ((y^4)^2 - (x^4)^2)$$

$$= xy (y^4 + x^4) (y^4 - x^4)$$

$$= xy (y^4+x^4) (y^2-x^2) (y^2+x^2)$$

$$= xy (y^4 + x^4) (y^2 + x^2) (y - x) (y + x)$$

19. Question

Factorize:

$$x^4 + x^2y^2 + y^4$$

Answer

Given,

$$x^4 + x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - x^2y^2$$

$$= (x^2y^2)^2 - (xy)^2$$

$$= (x^2 + y^2 - xy) (x^2 + y^2 + xy)$$

20. Question

Factorize:

$$x^2-y^2-4xz+4z^2$$

Answer

Given,

$$x^2 - y^2 - 4xz + 4z^2 = x^2 - 4xz + 4z^2 - y^2$$

$$= (x)^2 - 2 \times x \times 2z + (2z)^2 - y^2$$

$$= (x-2z)^2 - y^2$$

$$= (x - 2z - y) (x - 2z + y)$$

$$= (x - y - 2z) (x + y - 2z)$$

21. Question

Factorize:

$$x^2+6\sqrt{2}x+10$$

Answer

Given,

$$x^2+6\sqrt{2}x+10 = x^2+\sqrt{2}x + 10$$

$$= x(x+\sqrt{2}) + 5\sqrt{2} (x + \sqrt{2})$$

$$= (x+\sqrt{2}) (x+5\sqrt{2})$$

22. Question

Factorize:

$$x^2-2\sqrt{2}x-30$$

Answer

Given,

$$x^2 - 2\sqrt{2}x - 30 = x^2 - 5\sqrt{2}x + 3\sqrt{2}x - 30$$

$$= x(x-5\sqrt{2}) + 3\sqrt{2} (x - 5\sqrt{2})$$

$$= (x-5\sqrt{2}) (x+3\sqrt{2})$$

23. Question

Factorize:

$$x^2 - \sqrt{3} x - 6$$

Answer

Given,

$$X^2 - \sqrt{3}x - 6 = x^2 - 2\sqrt{3}x + \sqrt{3}x - 6$$

$$= x (x - 2\sqrt{3}) + \sqrt{3} (x-2\sqrt{3})$$

$$= (x + \sqrt{3}) (x - 2\sqrt{3})$$

24. Question

Factorize:

$$x^2+5\sqrt{5}x+30$$

Answer

Given,

$$x^2 + 5\sqrt{5}x + 30 = x^2 + 3\sqrt{5}x + 2\sqrt{5}x + 30$$

$$= x (x + 3\sqrt{5}) + 2\sqrt{5} (x + 3\sqrt{5})$$

$$= (x + 3\sqrt{5}) (x + 2\sqrt{5})$$

25. Question

Factorize:

$$x^2+2\sqrt{3}x-24$$

Answer





$$x^2 + 2\sqrt{3}x - 24 = x^2 + 4\sqrt{3}x - 2\sqrt{3}x - 24$$

$$= x(x + 4\sqrt{3}) - 2\sqrt{3} (x + 4\sqrt{3})$$

$$= (x + 4\sqrt{3}) (x - 2\sqrt{3})$$

Factorize:

$$2x^2 - \frac{5}{6}x + \frac{1}{12}$$

Answer

Given,

$$2x^2 - \frac{5}{6}x + \frac{1}{12}$$

$$=2x^2-\frac{1}{3}x-\frac{1}{2}x+\frac{1}{12}$$

$$=x\left(2x-\frac{1}{3}\right)-\frac{1}{4}\left(2x-\frac{1}{3}\right)$$

$$= \bigg(2x - \frac{1}{3}\bigg)\bigg(x - \frac{1}{4}\bigg)$$

27. Question

Factorize:

$$x^2 + \frac{12}{35}x + \frac{1}{35}$$

Answer

Given,

$$x^{2} - \frac{12}{36}x + \frac{1}{36} = x^{2} - \frac{1}{7}x - \frac{1}{5}x + \frac{1}{35}$$

$$x\left(x+\frac{1}{7}\right)+\frac{1}{5}\left(x+\frac{1}{7}\right)$$

$$=\left(x-\frac{1}{7}\right)\left(x+\frac{1}{5}\right)$$

28. Question

Factorize:

$$21x^2-2x+\frac{1}{21}$$

Answer

Given,

$$21x^2 - 2x + \frac{1}{21} = 21x^2 - x - x + \frac{1}{21}$$

$$= x(21x-1) - \frac{1}{21}(21x-1)$$

$$=\left(x-\frac{1}{21}\right)(21x-1)$$

29. Question

Factorize:



$$5\sqrt{5} x^2 + 20x + 3\sqrt{5}$$

Answer

Given,

$$5\sqrt{5}x^2+20x+3\sqrt{5} = 5\sqrt{5}x^2+15x+5x+3\sqrt{5}$$

$$= 5x(\sqrt{5}x+3) + \sqrt{5}(\sqrt{5}x+3)$$

$$= (5x + \sqrt{5}) (\sqrt{5}x + 3)$$

30. Question

Factorize:

$$2x^2+3\sqrt{5}x+531.9(2a-b)^2-4(2a-b)-13$$

Answer

Given,

$$2x^2+3\sqrt{5}x+5 = 2x^2 + 2\sqrt{5}x + \sqrt{5}x+5$$

$$= 2x (x + \sqrt{5}) + \sqrt{5} (x + \sqrt{5})$$

$$= (2x+\sqrt{5})(x+\sqrt{5})$$

31. Question

Factorize:

$$9(2a - b)^2 - 4(2a - b) - 13$$

Answer

Given,

$$9(2a - b)^2 - 4(2a - b) - 13$$

Let us assume (2a - b) = x

$$9x^2 - 4x - 13$$

$$9x^2 - 13x + 9x - 13$$

$$x(9x - 13) + 1(9x + 3)$$

$$(9x - 13)(x + 1)$$

$$(18a - 9b - 13)(2a - b + 1)$$

32. Question

Factorize:

$$7(x-2y)^2-25(x-2y)+12$$

Answer

Given,

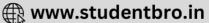
$$7(x-2y)^2-25(x-2y)+12$$

Let
$$a = (x - 2y)$$
,

So we have,

$$= 7a^2 - 25a + 12$$





$$= 7a^2 - 21a - 4a + 12$$

$$= 7a(a - 3) - 4(a - 3)$$

$$= (7a - 4) (a - 3)$$

Put
$$a = (x - 2y)$$

$$= \{7(x-2y)-4\} (x-2y-3)$$

$$= (7x - 14y - 4) (x - 2y - 3)$$

Factorize:

$$2(x+y)^2-9(x+y)-5$$

Answer

Given,

$$2(x+y)^2 - 9(x+y) - 5$$

$$= 2a^2 - 9a - 5$$

Let
$$(x+y) = a$$

$$= 2a^2 - 10a + a - 5$$

$$= 2a (a-5) + 1(a-5)$$

$$= (2a+1) (a-5)$$

$$= \{2(x+y)+1\}(x+y-5)$$

$$(2x+2y+1)(x+y-5)$$

34. Question

Give possible expressions for the length and breadth of the rectangle having $35y^2+13y-12$ as its area.

Answer

We know that,

Area of rectangle = length \times breadth

Given,

$$35y^2 + 13y - 12 = 35y^2 + 28y - 15y - 12$$

$$= 7y(5y+4) -3 (5y + 4)$$

$$= (7y - 3) (5y + 4)$$

Thus,

Length = (7y - 3), then breadth = (5y + 4)

Length = (5y + 4), then breadth = (7y - 3)

35. Question

What are the possible expressions for the dimensions of the cuboid whose volume is $3x^2-12x$.

Answer

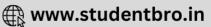
Given,

We know that,

Volume of cuboids = length \times breadth \times height







Given,

$$3x^2 - 12x = 3x (x-4)$$

Thus,

Dimensions of cuboids are -

Length	breadth	height
3	x	(x-4)
X	(x-4)	3
(x-4)	3	X

Exercise 5.2

1. Question

Factorize each of the following expressions:

$$p^{3}+27$$

Answer

Given,

$$P^3+27$$
,

$$= p^3 + (3)^3 [\because a^3 + b^3 = (a+b)(a^2 - 2ab + b^2)$$

$$= (p + 3) (p^2 + 9 - 3p)$$

2. Question

Factorize each of the following expressions:

$$y^3 + 125$$

Answer

Given,

$$y^3+125$$
,

$$= y^3 + (5)^3 [\because a^3 + b^3 = (a+b)(a^2 - 2ab + b^2)]$$

$$= (y + 5) (y^2 - 5y + 25)$$

3. Question

Factorize each of the following expressions:

$$1-27a^{3}$$

Answer

Given,

$$1 - 27a^3$$
,

$$= 1 - (3a)^3$$

$$= (1 - 3a) (1+9a^2+3a)$$

4. Question

Factorize each of the following expressions:

$$8x^3y^3+27a^3$$

Answer





Given,

$$8x^3y^3 + 27a^3$$
,

$$= (2xy)^3 + (3a)^3$$

$$= (2xy + 3a) (4x^2 y^2 + 9a^2 - 6axy)$$

5. Question

Factorize each of the following expressions:

Answer

Given,

$$64a^3 - b^3$$
.

$$= (4a)^3 - (b)^3$$

$$= (4a - b) (16a^2 + b^2 + 4ab)$$

6. Question

Factorize each of the following expressions:

$$\frac{x^3}{216}$$
-8y³

Answer

Given,

$$\frac{x^3}{216} - 8y^3 = \left(\frac{x}{6}\right)^3 - (2y)^3$$

$$= \Big(\frac{x}{6}-2y\Big) \bigg(\frac{x^2}{36}+4y^2+\frac{xy}{3}\bigg)$$

7. Question

Factorize each of the following expressions:

$$10x^4y-10xy^4$$

Answer

Given,

$$10x^4 y - 10xy^4$$

$$= 10xy (x^3 - y^3)$$

$$= 10xy (x - y) (x^2 + xy - y^2)$$

8. Question

Factorize each of the following expressions:

$$54x^6v + 2x^3v^4$$

Answer

$$54x^6y + 2x^3y^4$$

$$= 2x^3 y (27x^3 + y^3)$$





$$=2x^3 y \{(3x)^3+(y)^3\}$$

$$= 2x^3 y (3x + y) (9x^2 + y^2 - 3xy)$$

Factorize each of the following expressions:

$$32a^2 + 108b^3$$

Answer

Given,

$$32a^3 + 108b^3$$
,

$$= 4 (8a^3 + 27b^3)$$

$$= 4 \{ (2a)^3 + (3b)^3 \}$$

$$= 4 (2a + 3b) (4a^2 + 9b^2 - 6ab)$$

10. Question

Factorize each of the following expressions:

$$(a-2b)^3-512b^3$$

Answer

Given,

$$(a - 2b)^3 - 512b^3$$

$$= (a - 2b)^3 - (8b)^3$$

=
$$(a - 2b - 8b) \{(a - 2b)^2 + (8b)^2 + (a - 2b) 8b\}$$

$$= (a - 10b) (a^2 + 4b^2 - 4ab + 64b^2 + 8ab - 16b^2)$$

$$= (a - 10b) (a^2 + 52b^2 + 4ab)$$

11. Question

Factorize each of the following expressions:

$$(a+b)^3-8(a-b)^3$$

Answer

Given,

$$(a + b)^3 - \{2(a - b)\}^3$$

=
$$\{(a + b) - 2(a - b)\}\$$
 $\{(a+b)^2 + 4(a-b)^2 + 2(a+b)(a - b)\}\$ [By using: $x^3 - y^3 = (x - y)(x^2 + y^2 + xy)$]

$$= (a + b - 2a + 2b)(a^2 + b^2 + 2ab + 4a^2 + 4b^2 - 8ab + 2a^2 - 2b^2)$$

$$= (3b - a) (7a^2 + 3b^2 - 6ab)$$

12. Question

Factorize each of the following expressions:

$$(x+2)^3+(x-2)^3$$

Answer

$$(x+2)^3 + (x-2)^3 = (x+2+x-2) \{ (x+2)^2 + (x-2)^2 - (x+2)(x-2) \}$$





$$= 2x (x^2 + 4 + 4x + x^2 + 4 - 4x - x^2 + 4)$$

$$= 2x (x^2 + 12)$$

Factorize each of the following expressions:

$$8x^2y^3-x^5$$

Answer

Given,

$$8x^2y^3 - x^5$$

$$= x^2 (8y^3 - x^3)$$

$$= x^2 \{ (2y)^2 - (x)^3 \}$$

$$= x^2 (2y - x) (4y^2 + x^2 + 2xy)$$

14. Question

Factorize each of the following expressions:

$$1029-3x^3$$

Answer

Given,

$$1029 - 3x^3$$

$$= 3 (343 - x^3)$$

$$= 3 \{ (7)^3 - (x)^3 \}$$

$$= 3 (7 - x) (49 + x^2 + 7x)$$

15. Question

Factorize each of the following expressions:

$$x^{6}+y^{6}$$

Answer

Given,

$$X^6 + y^6 = (x^2)^3 + (y^2)^3$$

$$= (x^2 + y^2) (x^4 + y^4 - x^2y^2)$$

16. Question

Factorize each of the following expressions:

$$x^{3}y^{3}+1$$

Answer

Given,

$$X^3y^3 + 1 = (xy)^3 + (1)^3$$

$$= (xy + 1) (x^2 y^2 + 1 - xy)$$

17. Question

Factorize each of the following expressions:



$$x^4y^4-xy$$

Answer

Given,

$$X^4y^4 - xy = xy (x^3y^3 - 1)$$

$$= xy \{ (xy)^3 - (1)^3 \}$$

$$= xy (xy -1) (x^2y^2 +1+xy)$$

18. Question

Factorize each of the following expressions:

$$a^{12}+b^{12}$$

Answer

Given,

$$a^{12} + b^{12} = (a^4)^3 + (b^4)^3$$

$$= (a^4 + b^4)(a^3 + b^3 - a^4b^4)$$

19. Question

Factorize each of the following expressions:

$$x^3+6x^2+12x+16$$

Answer

Given,

$$X^3 + 6x^2 + 12x + 16 = (x^3 + 6x^2 + 12x + 8) + 8$$

$$= (x+2)^3 + 8 [\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$$

$$=(x+2)^3+(2)^3$$

$$= (x+2+2) \{(x+2)^2 +4 -2 (x+2)\}$$

$$= (x+4)(x^2+4+4x+4-2x-4)$$

$$= (x+4)(x^2+2x+4)$$

20. Question

Factorize each of the following expressions:

$$a^{3}+b^{3}+a+b$$

Answer

Given.

$$a^3+b^3+a+b = (a^3 + b^3) + (a+b)$$

$$= (a+b) (a^2 - ab + b^2) + a+b$$

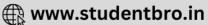
$$= (a+b)(a^2 - ab + b^2 + 1)$$

21. Question

Factorize each of the following expressions:

$$a^3 - \frac{1}{a^3} - 2a \frac{2}{a}$$





Answer

Given,

$$a^{3} - \frac{1}{a^{3}} - 2a + \frac{2}{a}$$

$$= a^{3} - \frac{1}{a^{3}} - 2\left(a - \frac{1}{a}\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^{2} + \frac{1}{a^{2}} + 1\right) - 2\left(a - \frac{1}{a}\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^{2} + \frac{1}{a^{2}} + 1 - 2\right)$$

$$= \left(a - \frac{1}{a}\right)\left(a^{2} + \frac{1}{a^{2}} - 1\right)$$

22. Question

Factorize each of the following expressions:

$$a^3+3a^2b^3+3ab^2+b^3-8$$

Answer

Given,

$$A^{3}+3a^{2}b+3ab^{2}+b^{3}-8 = (a^{3}+3a^{2}b+3ab^{2}+b^{3}) - 8$$

$$= (a+b)^{3} - 8$$

$$= (a+b)^{3} - (2)^{3}$$

$$= (a+b-2) \{ (a+b)^{2} + (2)^{2} + 2(a+b) \}$$

$$= (a+b-2) (a^{2}+b^{2} + 2ab+4+2a+2b)$$

23. Question

Factorize each of the following expressions:

$$8a^{3}-b^{3}-4ax+2bx$$

Answer

Given,

$$8a^{3} - b^{3} - 4 ax + 2bx = (2a)^{3} - (b)^{3} - 2x(2a - b)$$

$$= (2a - b) (4a^{2} + b^{2} + 2ab) - 2x(2a - b)$$

$$= (2a - b) (4a^{2} + b^{2} + 2ab - 2x)$$

24. Question

Simplify

$$\text{(i)}\, \frac{173\times173\times173+127\times127\times127}{173\times173-173\times127+127\times127}$$

$$(ii)\frac{155 \times 155 \times 155 - 55 \times 55 \times 55}{155 \times 155 + 155 \times 55 + 55 \times 55}$$

(iii)
$$\frac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 + 1.2 \times 0.2 + 0.2 \times 0.2}$$

Answer

(i) Given,



$$= \frac{173 \times 173 \times 173 + 127 \times 127 \times 127}{173 \times 173 - 173 \times 127 + 127 \times 127}$$

$$= \frac{(173)^3 + (127)^3}{(173)^2 - 173 \times 127 + (127)^2}$$

$$= \frac{(173 + 127)\{(173)^2 + (127)^2 - 173 \times 127\}}{(173)^2 - 173 \times 127 + (127)^2} = 300$$

(ii) Given,

$$=\frac{(155)^3-(55)^3}{(155)^2+155\times55+(55)^2}$$

$$=\frac{(155-55)\{(155)^2+155\times55+(55)^2\}}{(155)^2+155\times55+(55)^2}$$

$$=(155-55)=100$$

(ii) Given,

$$= \frac{1.2 \times 1.2 \times 1.2 - 0.2 \times 0.2 \times 0.2}{1.2 \times 1.2 + 1.2 \times 0.2 + 0.2 \times 0.2}$$

$$=\frac{(1.2)^3-(0.2)^3}{(1.2)^2+1.2\times0.2+(0.2)^2}$$

$$=\frac{(1.2-0.2)\{(1.2)^2+(0.2)^2+1.2\times0.2\}}{(1.2)^2+1.2\times0.2\times(0.2)^2}$$

$$=(1.2-0.2)=1$$

Exercise 5.3

1. Question

Factorize:

$$64a^3+125b^3+240a^2b+300ab^2$$

Answer

Given,

$$64a^3 + 125b^3 + 240a^2b + 300ab^2$$

$$= (4a)^3 + (5b)^3 + 60ab (4a + 5b)$$

$$= (4a)^3 + (5b)^3 + 3 \times 4a \times 5b (4a + 5b)$$

$$= (4a+5b)^3$$

$$= (4a+5b)(4a+5b)(4a+5b)$$

2. Question

Factorize:

$$125x^3 - 27y^3 - 225x^2y + 125xy^2$$

Answer

$$125x^3-27y^3-225x^2y+125xy^2$$
,





$$= (5x)^3 - (3y)^3 - 45xy (5x - 3y)$$

$$= (5x)^3 - (3y)^3 - 3 \times 5x \times 3y (5x - 3y)$$

$$= (5x - 3y)^3$$

$$= (5x - 3y) (5x - 3y) (5x - 3y)$$

Factorize:

$$\frac{8}{27}x^3+1+\frac{4}{3}x^2+2x$$

Answer

Given,

$$\frac{8}{27}x^3 + 1 + \frac{4}{3}x^2 + 2x$$

$$= \left(\frac{2}{3}x\right)^3 + (1)^3 + 2x\left(\frac{2}{3}x + 1\right)$$

$$= \left(\frac{2}{3}x\right)^3 + (1)^3 + 3 \times \frac{2}{3}x \times 1\left(\frac{2}{3}x + 1\right)$$

$$=\left(\frac{2}{3}x+1\right)^3$$

$$= \Big(\frac23x+1\Big)\Big(\frac23x+1\Big)\Big(\frac23x+1\Big)$$

4. Question

Factorize:

$$8x^3 + 27y^3 + 36x^2y + 54xy^2$$

Answer

Given,

$$8x^3+27y^3+36x^2y+54xy^2$$

$$= (2x)^3 + (3y)^3 + 18xy (2x + 3y)$$

$$= (2x)^3 + (3y)^3 + 3 \times 2x \times 3y (2x + 3y)$$

$$= (2x+3y)^3$$

$$= (2x +3y) (2x +3y) (2x +3y)$$

5. Question

Factorize:

$$a^{3}-3a^{2}b+3ab^{2}-b^{3}+8$$

Answer

$$a^3 - 3a^2b + 3ab^2 - b^3 + 8$$

$$= \{(a)^3 - (b)^3 - 3ab (a-b)\} + 8$$

$$= (a-b)^3 + (2)^3$$



=
$$(a - b + 2) \{(a - b)^2 + (2)^2 + 2(a - b)\}$$

$$= (a - b + 2)(a^2 - 2ab + b^2 + 4 + 2a - 2b)$$

Factorize:

$$x^3+8y^3+6x^2y+12xy^2$$

Answer

Given,

$$x^3+8y^3+6x^2y+12xy^2$$

$$= (x)^3 + (2y)^3 + 6xy (x + 2y)$$

$$= (x)^3 + (2y)^3 + 3 \times x \times 2y (x + 2y)$$

$$= (x+2y)^3$$

$$= (x +2y) (x +2y) (x +2y)$$

7. Question

Factorize:

$$8x^3+y^3+12x^2y+6xy^2$$

Answer

Given,

$$8x^3+y^3+12x^2y+6xy^2$$

$$= (2x)^3 + (y)^3 + 6xy (2x + y)$$

$$= (2x)^3 + (y)^3 + 3 \times 2x \times y (2x + y)$$

$$= (2x+y)^3$$

$$= (2x +y) (2x +y) (2x +y)$$

8. Question

Factorize:

$$8a^3 + 27b^3 + 36a^2b + 54ab^2$$

Answer

Given,

$$8a^3 + 27b^3 + 36a^2b + 54ab^2$$

$$= (2a)^3 + (3b)^3 + 18ab (2a + 3b)$$

$$= (2a)^3 + (3b)^3 + 3 \times 2a \times 3b (2a + 3b)$$

$$= (2a + 3b)^3$$

$$= (2a +3b) (2a +3b) (2a +3b)$$

9. Question

Factorize:

$$8a^3-27b^3-36a^2b+54ab^2$$

Answer





Given,

$$8a^3 - 27b^3 - 36a^2b + 54ab^2$$
,

$$= (2a)^3 - (3b)^3 - 18ab (2a - 3b)$$

$$= (2a)^3 - (3b)^3 - 3 \times 2a \times 3b (2a - 3b)$$

$$= (2a - 3b)^3$$

10. Question

Factorize:

$$x^3-12x(x-4)-64$$

Answer

Given,

$$x^3-12x(x-4)-64$$
,

$$= (x)^3 - 12x (x - 4) - (4)^3$$

$$= (x)^3 - (4)^3 - 3 \times x \times 4 (x - 4)$$

$$= (x - 4)^3$$

$$= (x - 4) (x - 4) (x - 4)$$

11. Question

Factorize:

$$a^{3}x^{3}-3a^{2}bx^{2}+3ab^{2}x-b^{3}$$

Answer

Given,

$$a^{3}x^{3}-3a^{2}bx^{2}+3ab^{2}x-b^{3}$$
.

$$= (ax)^3 - 3abx(ax-b) - (b)^3$$

$$= (ax)^3 - (b)^3 - 3abx (ax-b)$$

$$= (ax-b)^3$$

$$=$$
 (ax-b) (ax-b) (ax-b)

Exercise 5.4

1. Question

Factorize each of the following expressions:

$$a^3 + 8b^3 + 64c^3 - 24abc$$

Answer

Given,

$$= a^3 + 8b^3 + 64c^3 - 24abc$$

This can be written in form

$$= a^3 + 8b^3 + 64c^3 - 24abc = (a)^3 + (2b)^3 + (4c)^3 - 3 \times a \times 2b \times 4c$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula





Hence,

$$= a^3 + 8b^3 + 64c^3 - 24abc = (a + 2b + 4c)\{(a)^2 + (2b)^2 + (4c)^2 - a \times 2b - 2b \times 4c - 4c \times a\}$$

$$= (a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$$

Thus the required factors of $a^3 + 8b^3 + 64c^3 - 24abc$ is $(a + 2b + 4c)(a^2 + 4b^2 + 16c^2 - 2ab - 8bc - 4ca)$

2. Question

Factorize each of the following expressions:

$$x^3-8y^3+27z^3+18xyz$$

Answer

Given,

$$= x^3 - 8y^3 + 27z^3 + 18xyz$$

This can be written in form,

$$= x^3 - 8y^3 + 27z^3 + 18xyz = (x)^3 + (-2y)^3 + (3z)^3 - 3xxx(-2y)x3z$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= x^3 - 8y^3 + 27z^3 + 18xyz = \{x + (-2y) + 3z\}\{(x)^2 + (-2y)^2 + (3z)^2 - (x)(-2y) - (-2y)(3z) - (3z)(x)\}$$

$$= (x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$$

Thus the required factors of $x^3 - 8y^3 + 27z^3 + 18xyz$ is $(x - 2y + 3z)(x^2 + 4y^2 + 9z^2 + 2xy + 6yz - 3zx)$

3. Question

Factorize each of the following expressions:

$$27x^3-y^3-z^3-9xyz$$

Answer

Given,

$$= 27x^3 - y^3 - z^3 - 9xyz$$

This can be written in form,

$$= 27x^3 - y^3 - z^3 - 9xyz = (3x)^3 + (-y)^3 + (-z)^3 - 3(3x)(-y)(-z)$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

So,

$$= 27x^3 - y^3 - z^3 - 9xyz = \{3x + (-y) + (-z)\}\{(3x)^2 + (-y)^2 + (-z)^2 - (3x)(-y) - y(-z) - z(3x)\}$$

$$=(3x-y-z)(9x^2+y^2+z^2+3xy-yz+3zx)$$

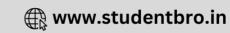
Thus the factors of $27x^3 - y^3 - z^3 - 9xyz$ is $(3x - y - z)(9x^2 + y^2 + z^2 + 3xy - yz + 3zx)$

4. Question

Factorize:

$$\frac{1}{27}x^3-y^3+125z^3+5xyz$$

Answer



$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz$$

This can be written in form,

$$= \frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz = \left(\frac{1}{3}x\right)^3 + (-y)^3 + (5z)^3 - 3\left(\frac{1}{3}x\right)(-y)(5z)$$

$$=a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= \frac{\frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz = \left\{ \left(\frac{1}{3}x \right) + (-y) + (5z) \right\} \left\{ \left(\frac{1}{3}x \right)^2 + (-y)^2 + (5z)^2 - \left(\frac{1}{3}x \right) (-y) - y \right\} (5z) - (5z) \left(\frac{1}{3}x \right)$$

$$= \left(\frac{1}{3}x - y + 5z\right) \left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$$

Thus the factors of
$$\frac{1}{27}$$
 $x^3 - y^3 + 125z^3 + 5xyz$ is $\left(\frac{1}{3}x - y + 5z\right)\left(\frac{1}{9}x^2 + y^2 + 25z^2 + \frac{1}{3}xy + 5yz - \frac{5}{3}zx\right)$

5. Question

Factorize each of the following expressions:

$$8x^3 + 27y^3 - 216z^3 + 108xyz$$

Answer

Given,

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz$$

This can be written in form,

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz = (2x)^3 + (3y)^3 + (-6z)^3 - 3(2x)(3y)(-6z)$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= 8x^3 + 27y^3 - 216z^3 + 108xyz = \{(2x) + (3y) + (-6z)\}\{(2x)^2 + (3y)^2 + (-6z)^2 - (2x)(3y) - (3y)((-6z) - (-6z)(2x)\}$$

$$= (2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$$

Thus the factors of

$$8x^3 + 27y^3 - 216z^3 + 108xyz$$
 is $(2x + 3y - 6z)(4x^2 + 9y^2 + 36z^2 - 6xy + 18yz + 12zx)$

6. Ouestion

Factorize each of the following expressions:

$$125 + 8x^3 - 27y^3 + 90xy$$

Answer

Given,

$$= 125 + 8x^3 - 27y^3 + 90xy$$

This can be written in form,

$$= 125 + 8x^3 - 27y^3 + 90xy = (5)^3 + (2x)^3 + (-3y)^3 - 3(5)(2x)(-3y)$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= \frac{125 + 8x^3 - 27y^3 + 90xy}{(2x)(-3y) - (-3y)(5)} = \frac{125 + 8x^3 - 27y^3 + 90xy}{(5x^2 + 2x^2 - 3y)(5)^2 + (2x)^2 + (-3y)^2 - (5)(2x) - (2x)(-3y)(5)}$$

$$= (5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$$

Thus the factors of $125 + 8x^3 - 27y^3 + 90xy$ is $(5 + 2x - 3y)(25 + 4x^2 + 9y^2 - 10x + 6xy + 15y)$







Factorize:

$$(3x-2y)^3+(2y-4z)^3+(4z-3x)^3$$

Answer

Given,

$$= (3x-2y)^3 + (2y-4z)^3 + (4z-3x)^3$$

Let
$$a = (3x - 2y)$$
, $b = (2y - 4z)$, $c = (4z - 3x)$

$$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = a^3 + b^3 + c^3$$

Here,

$$= a + b + c = (3x - 2y + 2y - 4z + 4z - 3x) = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

Hence,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (3x - 2y)^3 + (2y - 4z)^3 + (4z - 3x)^3 = 3(3x - 2y)(2y - 4z)(4z - 3x)$$

8. Question

Factorize each of the following expressions:

$$(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$$

Answer

Given,

$$=(2x-3y)^3+(4z-2x)^3+(3y-4z)^3$$

Let
$$a = (2x - 3y)$$
, $b = (4z - 2x)$, $c = (3y - 4z)$

Then,

$$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = a^3 + b^3 + c^3$$

Here,

$$= a + b + c = 2x - 3y + 4z - 2x + 3y - 4z = 0$$

Hence

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (2x - 3y)^3 + (4z - 2x)^3 + (3y - 4z)^3 = 3(2x - 3y)(4z - 2x)(3y - 4z).$$

9. Question

Factorize each of the following expressions:

$$\left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + y\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3$$

Answer



$$= \left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3$$

Let
$$a = \left(\frac{x}{2} + y + \frac{z}{3}\right)$$
, $b = \left(\frac{x}{3} - \frac{2y}{3} + z\right)$, $c = \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)$

Then,

$$= \left(\frac{x}{2} + y + \frac{z}{3}\right)^3 + \left(\frac{x}{3} - \frac{2y}{3} + z\right)^3 + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right)^3 = a^3 + b^3 + c^3$$

Here,

$$= a + b + c = \left(\frac{x}{2} + y + \frac{z}{3}\right) + \left(\frac{x}{3} - \frac{2y}{3} + z\right) + \left(-\frac{5x}{6} - \frac{y}{3} - \frac{4z}{3}\right) = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

Hence,

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$=\left(\frac{x}{2}+y+\frac{z}{3}\right)^3+\left(\frac{x}{3}-\frac{2y}{3}+z\right)^3+\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right)^3\\ =3\left(\frac{x}{2}+y+\frac{z}{3}\right)\left(\frac{x}{3}-\frac{2y}{3}+z\right)\left(-\frac{5x}{6}-\frac{y}{3}-\frac{4z}{3}\right).$$

10. Question

Factorize each of the following expressions:

$$(a-3b)^3+(3b-c)^3+(c-a)^3$$

Answer

Given,

$$=(a-3b)^3+(3b-c)^3+(c-a)^3$$

Let
$$x = (a-3b), v = (3b-c), z = (c-a)$$

Then,

$$= (a-3b)^3 + (3b-c)^3 + (c-a)^3 = x^3 + v^3 + z^3$$

Here,

$$= x + y + z = a - 3b + 3b - c + c - a = 0$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

Hence

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (a-3b)^3 + (3b-c)^3 + (c-a)^3 = 3(a-3b)(3b-c)(c-a)$$

11. Ouestion

Factorize each of the following expressions:

$$2\sqrt{2} a^3 + 3\sqrt{3} b^3 + c^3 - 3\sqrt{6} abc$$

Answer

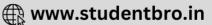
Given,

$$= 2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc$$

This can be written in form,







$$=2\sqrt{2}a^3+3\sqrt{3}b^3+c^3-3\sqrt{6}abc=\left(\sqrt{2}a\right)^3+\left(\sqrt{3}b\right)^3+(c)^3-3\left(\sqrt{2}a\right)\left(\sqrt{3}b\right)(c)$$

And,
$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

Hence,

$$= \frac{2\sqrt{2}a^3 + 3\sqrt{3}b^3 + c^3 - 3\sqrt{6}abc = \left\{ \left(\sqrt{2}a\right) + \left(\sqrt{3}b\right) + c \right\} \right\} \left\{ \left(\sqrt{2}a\right)^2 + \left(\sqrt{3}b\right)^2 + (c)^2 - \left(\sqrt{2}a\right)\left(\sqrt{3}b\right) - \left(\sqrt{3}b\right)(c) - (c)\left(\sqrt{2}a\right) \right\}$$

$$=\sqrt{2}a + \sqrt{3}b + c$$
) $(2a^2 + 3b^2 + c^2 - \sqrt{6}ab - \sqrt{3}bc - \sqrt{2}ca$

12. Question

Factorize each of the following expressions:

$$3\sqrt{3} a^3 - b^3 - 5\sqrt{5} c^3 - 3\sqrt{15} abc$$

Answer

Given,

$$= 3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc$$

This can be written in form .

$$=3\sqrt{3}a^3-b^3-5\sqrt{5}c^3-3\sqrt{15}abc=\left(\sqrt{3}a\right)^3+\left(-b\right)^3+\left(-\sqrt{5}c\right)^3-3\left(\sqrt{3}a\right)\left(-b\right)\left(-\sqrt{5}c\right)$$

And,
$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= \frac{3\sqrt{3}a^3 - b^3 - 5\sqrt{5}c^3 - 3\sqrt{15}abc = \{(\sqrt{3}a) + (-b) + (-\sqrt{5}c)\}\{(\sqrt{3}a)^2 + (-b)^2 + (-\sqrt{5}c)^2 - (\sqrt{3}a)(-b) - b\}(-\sqrt{5}c) - (-\sqrt{5}c)(\sqrt{3}a)$$

$$=(\sqrt{3}a-b-\sqrt{5}c)(3a^2+b^2+5c^2+\sqrt{3}ab-\sqrt{5}bc+\sqrt{15}ca)$$

13. Question

Factorize each of the following expressions:

$$8x^3-125y^3+180xy+216$$

Answer

Given,

$$= 8x^3 - 125y^3 + 180xy + 216$$

This can be written in form,

$$=8x^3-125y^3+180xy+216=(2x)^3+(-5y)^3+6^3-3(2x)(-5y)(6)$$

And,
$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

Hence ,

$$= 8x^3 - 125y^3 + 180xy + 216 = \{2x + (-5y) + 6\}\{(2x)^2 + (-5y)^2 + 6^2 - (2x)(-5y) - (-5y)(6) - (6)(2x)\}$$

$$= (2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

Thus the factors of $8x^3 - 125y^3 + 180xy + 216$ is

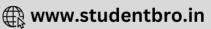
$$(2x - 5y + 6)(4x^2 + 25y^2 + 36 + 10xy + 30y - 12x)$$

14. Question

Factorize each of the following expressions:







$$2\sqrt{2} a^3 + 16\sqrt{2} b^3 + c^3 - 12abc$$

Answer

Given,

$$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$$

This can be written in form,

$$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = (\sqrt{2}a)^3 + (2\sqrt{2}b)^3 + c^3 - 3(\sqrt{2}a)(2\sqrt{2}b)(c)$$

And,
$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

Hence,

$$= 2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc = \{\sqrt{2}a + 2\sqrt{2}b + c\}\{(\sqrt{2}a)^2 + (2\sqrt{2}b)^2 + c^2 - (\sqrt{2}a)(2\sqrt{2}b - (2\sqrt{2}b)(c) - (c)(\sqrt{2}a)\}$$

$$= (\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca)$$

Thus the factors of $2\sqrt{2}a^3 + 16\sqrt{2}b^3 + c^3 - 12abc$ is $(\sqrt{2}a + 2\sqrt{2}b + c)(2a^2 + 8b^2 + c^2 - 4ab - 2\sqrt{2}bc - \sqrt{2}ca)$

15. Question

Find the value of $x^3+y^3-12xy+64$, when x+y=-4.

Answer

Given,

$$= x^3 + y^3 - 12xy + 64$$

$$= x + y = -4$$
 Given

$$= x+y+4 = 0$$

This can be written in form,

$$= x^3 + y^3 - 12xy + 64 = x^3 + y^3 + 4^3 - 3(x)(y)(4)$$

And ,
$$a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$
 By formula

$$= x^3 + y^3 - 12xy + 64 = (x + y + 4)\{(x)^2 + (y)^2 + 4^2 - xy - 4y - 4x\}$$

$$= 0 \times (x^2 + y^2 + 16 - xy - 4y - 4x)$$

= 0

16. Question

Multiply:

(i)
$$x^2+y^2+z^2-xy+xz+yzbyx+y-z$$

(ii)
$$x^2+4y^2+z^2+2xy+xz-2yzbyx-2y-z$$

(iii)
$$x^2+4y^2+2xy+-3x+6y+9byx-2y+3$$

(iv)
$$9x^2+25y^2+15xy+12x-20y+16by3x-5y+4$$

Answer

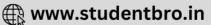
(i) Given,

$$= x^2 + v^2 + z^2 - xv + xz + vz$$

Multiply the above expression by (x + y - z)







$$= (x + y - z)(x^2 + y^2 + z^2 - xy + xz + yz)$$

$$= \{x + y + (-z)\}\{(x)^2 + (y)^2 + (-z)^2 - xy + yz + zx\}$$

$$= x^3 + y^3 + (-z)^3 - 3. x.y(-z)$$

$$= x^3 + y^3 - z^3 + 3xyz$$

(ii) Given,

$$= x^2 + 4y^2 + z^2 + 2xy + xz - 2yz$$

Multiply above expression by (x-2y-z)

Then,

$$=(x-2y-z)(x^2+4y^2+z^2+2xy+xz-2yz)$$

$$= \{x + (-2y) + (-z)\}\{x^2 + 4y^2 + z^2 + 2xy - 2yz + zx\}$$

By formula...

$$= x^3 - 8y^3 - z^3 - 6xyz$$

iii) we have

$$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$$

(iii) Given,

$$= x^2 + 4y^2 + 2xy - 3x + 6y + 9$$

Multiply above equation by (x-2y+3)

$$= (x-2y+3)(x^2+4y^2+2xy-3x+6y+9)$$

$$= \{x - 2y + 3\}\{x^2 + 4y^2 + 9 + 2xy + 6y - 3x\}$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= x^3 - 8y^3 + 27 + 18xy$$

(iv) Given,

$$= 9x^2 + 25y^2 + 15xy + 12x - 20y + 16$$

Multiply above equation by (3x - 5y + 4)

We got,

$$= (3x - 5y + 4)(9x^2 + 25y^2 + 15xy + 12x - 20y + 16) = (3x + (-5y) + 4)((3x)^2 + (-5y)^2 + 4^2 - 3x(-5y) - (-5y)(4) - (4)(3x))$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= (3x)^3 + (-5y)^3 + (4)^3 - 3.3x(-5y)(4)$$

$$= 27x^3 - 125y^3 + 64 + 180xy$$

CCE - Formative Assessment

1. Question

Factorize: x^4+x^2+25 .

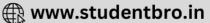
Answer

We have,

First we rewrite the question,







$$x^{4} + x^{2} + 25 = (x^{2})^{2} + 2.x^{2}.5 + 52 - 9x^{2}$$

$$= \{(x^{2})^{2} + 2.x^{2}.5 + 52\} - (3x)^{2} \text{ [By using } a^{2} + 2ab + b^{2} = (a + b)^{2}]$$

$$= \{x^{2} + 5\}^{2} - (3x)^{2} \text{ [By using } a^{2} - b^{2} = (a + b) (a - b)$$

$$= (x^{2} + 5 + 3x) (x^{2} + 5 - 3x)$$

Thus, the factors of $x^4 + x^2 + 25$ are $(x^2 + 5 + 3x)(x^2 + 5 - 3x)$.

2. Question

Factorize: $x^2-1-2a-a^2$.

Answer

We have,

$$= x^2 - 1 - 2a - a^2$$

Taking -1 as common from last three terms

$$= x^2 - (1 + 2a + a^2)$$

$$= x^2 - (1^2 + 2 \times 1 \times a + a^2)$$

$$= x^2 - (1+a)^2 [a^2 - b^2 = (a+b)(a-b)]$$

$$= (x+1+a)(x-1-a)$$

Thus the factors of $x^2 - 1 - 2a - a^2$ are (x + 1 + a)(x - 1 - a).

3. Question

If a + b + c = 0, then write the value of $a^3 + b^3 + c^3$.

Answer

We have,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

When (a + b + c) = 0 Given

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 - 3abc = 0$$

$$= a^3 + b^3 + c^3 = 3abc$$
.

4. Question

 $Ifa^2+b^2+c^2=20$,anda+b+c=0,findab+bc+ca.

Answer

We have,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
 By formula

$$= a^2 + b^2 + c^2 = 20$$
 Given

$$=$$
 $(a+b+c) = 0$ Given

Then.

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$= (0)^2 = 20 + 2(ab + bc + ca)$$





$$= 2(ab + bc + ca) = -20$$

$$= (ab + bc + ca) = -10$$

If
$$a + b + c = 9$$
 and $ab + bc + ca = 40$, find $a^2 + b^2 + c^2$.

Answer

We have,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
 By formula

$$= a + b + c = 9$$
 Given

$$= ab + bc + ca = 40$$
 Given

Then,

$$= (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$= (9)^2 = a^2 + b^2 + c^2 + 2 \times 40$$

$$= a^2 + b^2 + c^2 = 81 - 80$$

$$= a^2 + b^2 + c^2 = 1$$

6. Question

If
$$a^2 + b^2 + c^2 = 250$$
 and $ab + bc + ca = 3$, find $a + b + c$.

Answer

We have,

$$= (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$
 By formula

$$= a^2 + b^2 + c^2 = 250$$
 Given

$$= ab + bc + ca = 3$$
 Given

Then,

$$= (a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$= (a + b + c)^2 = 250 + 2 \times 3$$

$$= (a+b+c)^2 = 256$$

$$= (a+b+c) = \sqrt{256} = \pm 16$$
.

7. Question

Write the value of: $25^3 - 75^3 + 50^3$.

Answer

We have,

$$=25^3-75^3+50^3$$

Let
$$a = 25$$
, $b = -75$, $c = 50$,

Then the expression becomes as,

$$= 25^3 - 75^3 + 50^3 = a^3 + b^3 + c^3$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula





Here,
$$(a+b+c) = 25 + (-75) + 50 = 0$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= 25^3 - 75^3 + 50^3 = 3 \times 25 \times -75 \times 50$$

$$= 25^3 - 75^3 + 50^3 = -281250.$$

8. Question

Write the value of: $48^3 - 30^3 - 18^3$.

Answer

We have,

$$=48^3-30^3-18^3$$
 Given

Let
$$a = 48$$
, $b = -30$, $c = -18$

Then the expression becomes,

$$=48^3-30^3-18^3=a^3+b^3+c^3$$

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

Here,

$$= (a+b+c) = 48 + (-30) + (-18) = 0$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$=48^3-30^3-18^3=3\times48\times-30\times-18$$

$$=48^3-30^3-18^3=77760.$$

9. Question

Write the value of: $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$

Answer

We have,

$$=\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3$$
 Given

Let $a = \frac{1}{2}$, $b = \frac{1}{3}$, $c = -\frac{5}{6}$, then the expression becomes

$$=\left(\frac{1}{2}\right)^3+\left(\frac{1}{3}\right)^3-\left(\frac{5}{6}\right)^3=a^3+b^3+c^3$$

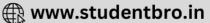
Here,

$$= a + b + c = \frac{1}{2} + \frac{1}{3} - \frac{5}{6} = 0$$

Hence,







$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = 3 \times \frac{1}{2} \times \frac{1}{3} \times -\frac{5}{6}$$

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = -\frac{5}{12}$$

Write the value of: $30^3 + 20^3 - 50^3$.

Answer

We have,

$$=30^3+20^3-50^3$$
 given

Let
$$a = 30$$
, $b = 20$, $c = -50$, then the expression becomes,

$$=30^3+20^3-50^3=a^3+b^3+c^3$$

Here,

$$= a + b + c = 30 + 20 - 50 = 0$$

Hence,

$$= a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
 By formula

$$= a^3 + b^3 + c^3 - 3abc = 0 \times (a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + b^3 + c^3 = 3abc$$

$$=30^3+20^3-50^3=3\times30\times20\times-50$$

$$=30^3+20^3-50^3=-90000$$

1. Question

The factors of a^2 -1-2x- x^2 are

A.
$$(a-x+1)(a-x-1)$$

B.
$$(a+x-1)(a-x+1)$$

C.
$$(a+x+1)(a-x-1)$$

D. none of these

Answer

We have,

$$= a^2 - 1 - 2x - x^2$$

$$= a^2 - (1 + 2x + x^2) = a^2 - (1 + x)^2$$

$$= (a-1-x)(a+1+x) [a^2-b^2 = (a+b)(a-b)]$$

Thus, the factors of $a^2 - 1 - 2x - x^2$ are (a - 1 - x)(a + 1 + x).

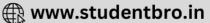
2. Question

The factors of $x^4+x^2=25$ are

A.
$$(x^2+3x+5)(x^2-3x+5)$$







B.
$$(x^2+3x+5)(x^2+3x-5)$$

C.
$$(x^2+x+5)(x^2-x+5)$$

D. none of these

Answer

We have.

$$= x^4 + x^2 + 25$$

Adding and subtracting $9x^2$ in the equation

$$=x^4 + 25 + x^2 + 9x^2 - 9x^2$$

$$= x^4 + 25 + 10x^2 - 9x^2 = (x^2 + 5)^2 - (3x)^2$$

$$=(x^2+5-3x)(x^2+5+3x)$$

Thus the factors of (x^4+x^2+25) are $(x^2+5-3x)(x^2+5-3x)$.

3. Ouestion

The factors of $x^2+4y^2+4y-4xy-2x-8$ are

A.
$$(x-2y-4)(x-2y+2)$$

B.
$$(x-y+2)(x-4y-4)$$

C.
$$(x+2y-4)(x+2y+2)$$

D. none of these

Answer

We have,

$$= x^2 + 4y^2 + 4y - 4xy - 2x - 8$$

$$= x^2 + 4y^2 - 4xy + 4y - 2x - 8 = (x - 2y)^2 - 2(x - 2y) - 8$$

Let a = (x-2y), then the expression becomes,

$$= x^2 + 4y^2 + 4y - 4xy - 2x - 8 = a^2 - 2a - 8$$

$$= a^2 - 4a + 2a - 8$$

$$= a(a-4) + 2(a-4)$$

$$=(a-4)(a+2)$$

Put
$$a = (x - 2y)$$

$$= x^{2} + 4y^{2} + 4y - 4xy - 2x - 8 = (x - 2y - 4)(x - 2y + 2)$$

Thus the factors of $x^2 + 4y^2 + 4y - 4xy - 2x - 8$ are (x - 2y - 4)(x - 2y + 2)

4. Question

The factors of $x^3-x^2y-xy^2+y^3$ are

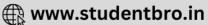
A.
$$(x+y)(x^2-xy+y^2)$$

B.
$$(x+y)(x^2+xy+y^2)$$

C.
$$(x+y)^2(x-y)$$

D.
$$(x-y)^2(x+y)$$





Answer

We have,

$$= x^3 - x^2y - xy^2 + y^3$$

$$= x^3 + y^3 - x^2y - xy^2 = (x + y)^3 - 3x^2y - 3xy^2 - x^2y - xy^2$$

As
$$x^3 + y^3 = (x + y)^3 - 3x^2y - 3xy^2$$

$$= (x + y)^3 - 4xy(x + y)$$

$$=(x + y) \{(x + y)^2 - 4xy\}$$

$$= (x + y)(x^2 + y^2 + 2xy - 2xy)$$

$$=(x + y)(x^2 + y^2 - 2xy) = (x + y)(x - y)^2$$

5. Question

The factors of x^3-1+y^3+3xy are

A.
$$(x-1+y)(x^2+1+y^2+x+y-xy)$$

B.
$$(x+y+1)(x^2+y^2+1-xy-x-y)$$

C.
$$(x-1+y)(x^2-1-y^2+x+y+xy)$$

D.
$$3(x+y-1)(x^2+y^2-1)$$

Answer

We have,

$$= x^3 - 1 + y^3 + 3xy$$

$$= (x)^3 + (-1)^3 + (y)^3 - 3(x)(-1)(y)$$

$$= (x-1+y)(x^2+1+y^2+x+y-xy) \{(a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)\}$$

Thus the factors of $x^3 - 1 + y^3 + 3xy$ are $(x - 1 + y)(x^2 + 1 + y^2 + x + y - xy)$

6. Question

The factors of 8a²+b³-6ab+1are

A.
$$(2a+b-1)(4a^2+b^2+1-3ab-2a)$$

B.
$$(2a-b+1)(4a^2+b^2-4ab+1-2a+b)$$

C.
$$(2a+b+1)(4a^2+b^2+1-2ab-b-2a)$$

D.
$$(2a-1+b)(4a^2+1-4a-b-2ab)$$

Answer

We have .

$$=8a^3+b^3-6ab+1$$

$$= (2a)^3 + (b)^3 - 3(2a)(b)(1) + (1)^3$$
 try to make $(a^3 + b^3 + c^3 - 3abc)$

$$= (2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$$

Thus the factors of $8a^3 + b^3 - 6ab + 1$ are $(2a + b + 1)(4a^2 + b^2 + 1 - 2ab - b - 2a)$.

7. Question

 $(x+y)^3$ - $(x-y)^3$ can be factorized as:







A.
$$2y(3x^2+y^2)$$

B.
$$2x(3x^2+y^2)$$

C.
$$2y(3y^2+x^2)$$

D.
$$2x(x^2+3y^2)$$

Answer

We have,

$$=(x+y)^3-(x-y)^3$$

Applying formulas,

$$= x^3 + y^3 + 3x^2y + 3xy^2 - (x^3 - y^3 - 3x^2y + 3xy^2)$$

$$= x^3 + y^3 + 3x^2y + 3xy^2 - x^3 + y^3 + 3x^2y - 3xy^2$$

$$= 2y^3 + 6x^2y$$

$$= 2y(y^2 + 3x^2)$$

Thus the factors of $(x + y)^3 - (x - y)^3$ are $2y(y^2 + 3x^2)$

8. Question

The factors of x^2 -7x+6are

A.
$$x(x-6)(x-1)$$

B.
$$(x^2-6)(x-1)$$

C.
$$(x+1)(x+2)(x-3)$$

D.
$$(x-1)(x+3)(x-2)$$

Answer

We have,

$$= x^3 - 7x - 6$$

Adding and subtracting 1 in the equation

$$= x^3 - 7x - 6 - 1 + 1 = x^3 + 1^3 - 7x - 7$$

$$= (x+1)(x^2+1+x)-7(x+1)$$

$$=(x+1)(x^2-6+x)$$

$$=(x+1)(x^3-3x+2x+x)=(x+1)(x+2)(x-3)$$

Thus the factors of $x^3 - 7x - 6$ are (x + 1)(x + 2)(x - 3).

9. Question

The expression $(a-b)^3+(b-c)^3+(c-a)^3$ can be factorized as:

C.
$$-3(a-b)(b-c)(c-a)$$

D.
$$(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

Answer

We have,



Let
$$x = (a - b), y = (b - c), z(c - a)$$

So,
$$x + y + z = a - b + b - c + c - a$$

If a+b+c=0, then,

$$= a^3 + b^3 + c^3 = 3abc$$

$$= (a-b)^3 + (b-c)^3 + (c-a)^3 = 3(a-b)(b-c)(c-a).$$

10. Question

The expression x^4+4 can be factorized as

A.
$$(x^2+2x+2)(x^2-2x+2)$$

B.
$$(x^2+2x+2)(x^2+2x-2)$$

C.
$$(x^2-2x-2)(x^2-2x+2)$$

D.
$$(x^2+2)(x^2-2)$$

Answer

We have,

$$= x^4 + 4$$

$$= x^4 + 4x^2 + 4 - 4x^2$$

$$=(x^2+2)^2-4x^2$$

$$= (x^2 + 2)^2 - (2x)^2 [x^2 - y^2 = (x - y)(x + y)]$$

$$=(x^2+2+2x)(x^2+2-2x)$$

11. Question

If 3x=a+b+c, then the value of $(x-a)^3+(x-b)^3+(x-c)^3-3(x-a)(x-b)(x-c)$ is

$$A. a+b+c$$

D. 0

D. none of these

Answer

We have,

$$= 3x = a+b+c$$

Let
$$a_1 = x - a$$
, $b_1 = x - b$, $c_1 = x - c$

So,
$$a_1 + b_1 + c_1 = x - a + x - b + x - c = 3x - (a + b + c)$$

$$= 3x - 3x = 0$$
 [a +b + c = 3x] given

$$= (x-a)^3 + (x-b)^3 + (x-c)^3 = 3(x-a)(x-b)(x-c)$$

Now,
$$(x-a)^3 + (x-b)^3 + (x-c)^3 - 3(x-a)(x-b)(x-c) = 0$$
.

12. Question

If
$$(x+y)^3-(x-y)^3-6y(x^2-y^2)=ky^2$$
 then $k=$

A. 1

B. 2



Answer

We have,

$$=(x+y)^3-(x-y)^3-6y(x^2-y^2)=ky^3$$

$$= (x+y-x+y)^3 + 3(x+y)(x-y)(x+y-x+y) - 6y(x^2-y^2) = ky^3$$

$$= 2y^3 + 6y(x^2 - y^2) - 6y(x^2 - y^2) = ky^3$$

$$= 8y^3 = ky^3$$

$$= k = 8$$
.

13. Question

 $Ifx^3-3x^2+3x-7=(x+1)(ax^2+bx+c)$, then a+b+c=

- A. 4
- B. 12
- C. -10
- D. 3

Answer

We have,

$$= x^3 - 3x^2 + 3x - 7 = (x + 1)(ax^2 + bx + c)$$

$$=x^3 - 3x^2 + 3x - 7 = ax^3 + bx^2 + cx + ax^2 + bx + c$$

$$= x^3 - 3x^2 + 3x - 7 = ax^3 + (a + b)x^2 + (b + c)x + c$$

By compairing both sides,

$$= a = 1$$

$$= a + b = -3$$

$$= b + c = 3$$

$$= c = -7$$

Thus, a + (b + c) = 1+3 = 4.

14. Question

The value of $\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}$ is

- A. 2
- B. 3
- C. 2.327
- D. 2.273

Answer

We have,

$$=\frac{2.3^3-0.027}{2.3^2+0.69+0.09}$$



$$= \frac{2.3^3 - 0.027}{2.3^2 + 0.69 + 0.09} = \frac{2.3^3 - 0.3^3}{2.3^2 + 0.3^2 + 2.3 \times 0.3} \left[a^3 - b^3 = (a - b)(a^2 + b^2 + ab) \right]$$

Hence,

$$= \frac{\{(a-b)(a^2+b^2+ab)\}}{a^2+b^2+ab} = a - b$$

$$= (2.3 - 0.3) = 2$$
.

15. Question

The value of $\frac{(0.013)^3+(0.007)^3}{(0.013)^2-0.013\times0.013\times0.007+(0.007)^2}is$

- A. 0.006
- B. 0.02
- C. 0.0091
- D. 0.00185

Answer

We have,

$$= \frac{{}^{0.013^3 + 0.007^3}}{{}^{0.013^2 - 0.013 \times 0.007 + 0.007^2}} [\ a^3 + b^3 = (a + b\)(a^2 + b^2 - ab)]$$

Hence,

$$= \frac{\{(a+b)(a^2+b^2-ab)\}}{a^2+b^2-ab} = a+b$$

$$= 0.013 + 0.007 = 0.020$$

